$$\text{Determiner } S_n = \sum_{p=0}^{n} \binom{n}{p} \cos(px)$$

$$\text{On pose } : \begin{cases} A_n = \displaystyle\sum_{p=0}^{n} \binom{n}{p} \cos(px) \\ B_n = \displaystyle\sum_{p=0}^{n} \binom{n}{p} \sin(px) \end{cases}$$

On a donc $A_n + iB_n = \displaystyle\sum_{p=0}^{n} \binom{n}{p} (\cos(px) + i\sin(px)) = \sum_{p=0}^{n} \binom{n}{p} (e^{ix})^p$ car $p \in \mathbb{N}$

$$\text{Et donc } \sum_{p=0}^{n} \binom{n}{p} (e^{ix})^p = (1+e^{ix})^n = e^{i\frac{nx}{2}}(e^{i\frac{x}{2}} + e^{-i\frac{x}{2}})^n$$
$$= 2^n \cos^n(\frac{x}{2})(\cos(\frac{nx}{2}) + i\sin(\frac{nx}{2}))$$

Donc par unicité de la partie réelle,
$S_n = Re\left(2^n \cos^n(\frac{x}{2})(\cos(\frac{nx}{2}) + i\sin(\frac{nx}{2}))\right) = 2^n \cos^n(\frac{x}{2})\cos(\frac{nx}{2})$
Ce qui conclut l'exercice.